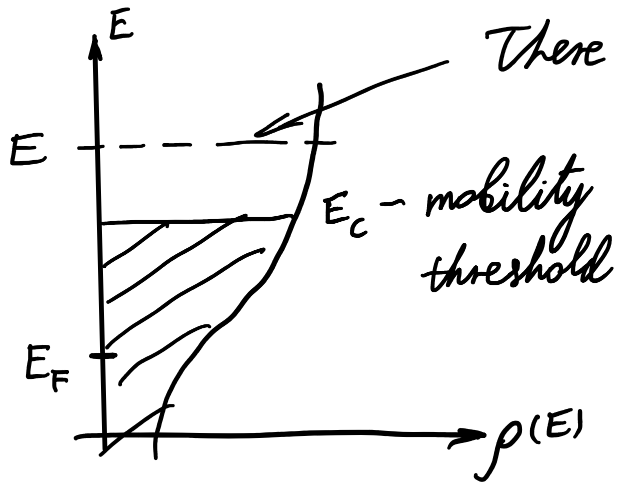
Hopping transport. Mott's law.

Consider an arbitrary system where the Fermi level is located among localised states



There is some finite concentration of electrons here

$f \sim e^{-\frac{E-E_F}{T}}$

Total conductivity

$$\sigma = \int dE \left(-\frac{\partial f}{\partial E}\right) \sigma(\varepsilon_F = E)$$

When $f = \theta(E_F - E)$, we just get $\sigma = \sigma(E_F)$

$\sigma(\varepsilon)$ – the „metallic" conductivity of a system with the Fermi level ε

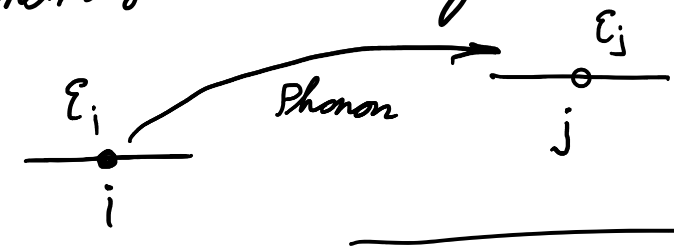
When E_F is deep inside localised states

$$\sigma \sim e^{-\frac{E_c - E_F}{T}}$$

Is there transport between localised and localised states? What if <u>all</u> states are localised?

In fact, they may absorb and emit

In fact, they may absorb and emit phonons inelastically



i and j - some localised states (e.g. states on impurities)

Let's assume all electrons are localised on impurities = attractive centres that host electron states

$$\exists\ \psi \propto \frac{1}{r} e^{-\frac{r}{a_B}}, \quad a_B = \frac{\varkappa \hbar^2}{m^* e^2}$$

$$\psi_i \sim e^{-\frac{|r-r_i|}{\xi}}, \quad \xi \text{ - localisation length}$$

Transition rate:

$$\frac{1}{\tau_{ij}} \propto F(\varphi_{ij}, f_i, f_j) \int |M_q|^2 \, \delta(\hbar q s - \varepsilon_{ij}) \, d^3q$$

Speed of sound

Energy gap between states

$$\varphi_{ij} = \frac{1}{e^{\frac{|\varepsilon_i - \varepsilon_j|}{T}} - 1}$$

One may assume that there exists a network of randomly located sites i and j, and there is an effective ... R_{ij} between them with

i and j, and ... is
resistance R_{ij} between them with

$$R_{ij} \propto e^{-\frac{2r_{ij}}{\xi} + \frac{\max(\varepsilon_i, \varepsilon_j)}{T}}$$

Assume $\varepsilon_i < \varepsilon_j$
Consider the rate of hopping $i \to j$
The probability to have an electron on site i is $f_i \sim e^{-\frac{\varepsilon_i}{T}}$, the probability to find a phonon which bridges energies accordingly is $\varphi_{ij} \sim e^{-\frac{\varepsilon_j - \varepsilon_i}{T}}$
Their product $\Gamma_{i \to j} \propto e^{-\frac{\varepsilon_j}{T}}$

The rate $j \to i$ is proportional to the probability f_j to have an electron on site j

$$\Gamma_{j \to i} \propto e^{-\frac{\varepsilon_j}{T}}$$

Hopping on nearest sites

$R \sim e^{\frac{A}{T}}$ where $A \sim$ typical ε_i

Variable-range hopping

Let's assume that hopping occurs through sites of energy ε

Let's assume ...

sites of energy ε

What is the density (typical distance) between such sites?

The concentration of sites with energies $\leq \varepsilon$ is $N(E<\varepsilon) = g\,\varepsilon$

The average distance between them is

$$\bar{r}_\varepsilon = N^{-\frac{1}{d}}_{(E<\varepsilon)} = (g\,\varepsilon)^{-\frac{1}{d}}$$

The typical resistance between sites is

$$R \sim \exp\left(-\frac{1}{\xi\,(g\,\varepsilon)^{\frac{1}{d}}} - \frac{\varepsilon}{T}\right)$$

// absorbed 2 into g

<u>Minimise the exponent</u>

Excitations, on the one hand, are trying to maximise the overlap integral, but, on the other hand, want to minimise the thermal activation energy.

$$-\frac{1}{d\,\xi\,g^{\frac{1}{d}}}\,\frac{1}{\varepsilon^{\frac{d+1}{d}}} + \frac{1}{T} = 0$$

$$\rightarrow \varepsilon^{\frac{d+1}{d}} = \frac{T}{d\,\xi\,g^{\frac{1}{d}}} \rightarrow \varepsilon^{\frac{1}{d}} = \left(\frac{T}{d\,\xi\,g^{\frac{1}{d}}}\right)^{\frac{1}{d+1}}$$

$$\rightarrow \varepsilon = \frac{\quad}{d \xi g^{\frac{1}{d}}} \rightarrow \quad \left(d \xi g^{d} \right)$$

The value of the exponent

$$A \frac{1}{\xi^{\frac{d}{d+1}} g^{\frac{1}{d+1}}} \frac{1}{T^{\frac{1}{d+1}}} = \left(\frac{T_0}{T}\right)^{\frac{1}{d+1}}$$

where $T_0 \sim \frac{1}{g \xi^d}$

$$\boxed{R \propto e^{-\left(\frac{T_0}{T}\right)^{\frac{1}{d+1}}}}$$